Reliability Analysis of Static Soil Liquefaction Using Random Finite Element Method

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Abstract
Liquefaction of soils, defined as significant reduction in shear strength and stiffness due to increase in pore pressure. This phenomenon can be assessed in static or dynamic loading types. However, in each type, the inherent variability of the soil parameters dictates that this problem is of a probabilistic nature rather than being deterministic. In this research, a random finite element analysis is used for reliability assessment of static liquefaction potential of loose sand under monotonic loading. The Monte Carlo simulation was used for that purpose. The selected stochastic parameters are soil parameters such as unit weight, peak friction angle and initial plastic shear modulus. An elasto-plastic effective stress model is used that simulates the static liquefaction response of loose sands under monotonic loading. The modified Newton-Raphson method is used to consider the effect of changing material behavior in this research. Analysis process was performed in MATLAB code.

Keywords: Static soil liquefaction, Random Finite Element Method, Monte Carlo simulation, Monotonic loading

1. INTRODUCTION
Liquefaction under monotonic undrained loading, commonly called ‘static liquefaction’, is typically associated with loose and very loose saturated sands and sand–silt mixtures while in situ and under relatively low stress conditions, and may be defined as a large reduction of mean effective pressure induced by a persistent generation of pore pressures. It has been popularly recognized that the liquefaction induced ground failures caused severe damage in various forms such as sand boiling, ground settlement, lateral spreading, landslide, etc. [1].

The first study of static liquefaction was performed by Castro [2], who analyzed the effects of initial void ratio on un-drained behavior of sand. Castro shows that loose sand, which presents a compactive behavior in drained conditions, exhibits, in undrained conditions, an increase of pore pressure leading to a deterioration of effective stresses. Lade and Yamamuro [3], investigated the effect of fines content on the static liquefaction potential under monotonic loading. The results clearly show that the presence of fines can greatly increase the potential for static liquefaction of clean sand. Prisco and Silvia [4], evaluated the static liquefaction of a saturated loose sand stratum with one dimension finite difference numerical code using an elasto-viscoplastic constitutive model. Wanatowski and Chu [5], studied the static liquefaction behavior of sand under undrained plain-strain conditions and a unique relationship between the stress ratio of the instability line and the state parameter are established to enable the triaxial results to be used for plain-strain conditions. Della et al. [6], a study conducted for identification of the behavior of the Chlef sand to static liquefaction. They show that the initial confining pressure and the relative density affect considerably the resistance to liquefaction. Ibraim et al. [7], studied the static liquefaction of fibre reinforced sand under monotonic loading.

In this paper, a computer program for evaluation static liquefaction potential of sandy soils, based on finite element method, also reliability assessment of safety factor against liquefaction was coded in MATLAB,
based on an elasto-plastic effective stress constitutive model developed by Byrne et al. [8]. This constitutive model is based on the characteristic behavior of the soil skeleton as observed in laboratory element tests. The controlling philosophy behind this model one of the simplicity, an avoidance of unnecessary complexity. The Monte Carlo simulation was used as reliability assessment approach.

2. ELASTO-PLASTIC EFFECTIVE STRESS CONSTITUTIVE MODEL

The numerical model used in this paper is based on plasticity theory and the characteristic sand behavior observed in laboratory tests under static and cyclic loading conditions. The model predicts the shear stress-strain behavior of the soil using an assumed hyperbolic relationship, and estimates the associated volumetric response of the soil skeleton using a flow rule that is a function of the current stress ratio. It is briefly presented in the following section:

2.1 ELASTIC RESPONSE

The elastic shear modulus, $G^e$, and bulk modulus, $B^e$ are assumed to be isotropic and stress level dependent. They are described by the following relations:

$$G^e = k^e_G P_a \left( \frac{P'_a}{P_a} \right)^{n_e}$$

(1)

$$B^e = k^e_B P_a \left( \frac{P'_a}{P_a} \right)^{n_e}$$

(2)

Where: $k^e_G$ and $k^e_B$ are modulus numbers, $P_a$ is atmospheric pressure, $P'_a$ is the mean effective stress $P'_a = \frac{\sigma' + \sigma''}{2}$, $n_e$ and $n_e$ are the elastic bulk and shear modulus exponent.

2.2 PLASTIC RESPONSE

Plastic shear strains are assumed to be caused by an increase in stress ratio $\Delta \eta$. An approach similar to that of Duncan and Chang [9] is adopted here, but modified as follows: (1) only the plastic component of shear strain is assumed to follow a hyperbolic formulation; and (2) the plastic shear strain is controlled by the stress ratio $\eta$ rather than the shear stress only.

$$\Delta \gamma^p = \left( \frac{1}{G^*} \right) \Delta \eta_d$$

(3)

$$G^* = \frac{G^p}{P'} \left[ 1 - \left( \frac{\eta_d}{\eta_f} R_f \right) \right]^2$$

(4)

Where:

$G^*$, $\Delta \gamma^p$ and $\Delta \eta_d$ are the normalized tangent plastic shear modulus, the plastic shear strain increment, and the developed stress ratio increment, respectively. $k^p_G$ is the plastic shear modulus number; $n_p$ is the plastic shear modulus exponent; $\eta$ is the stress ratio at failure = $(q/p')_f = \sin \phi_f$, where $\phi_f$ is the peak friction angle; and $R_f$ is the failure ratio $= \eta_f / \eta_u$, generally ranging from 0.5 to 1.0, where $\eta_u$ is the ultimate strength from the best fit hyperbola.

2.3 FLOW RULE

The plastic volumetric strain increment $\Delta \varepsilon^p_v$ is obtained from a flow rule based on energy considerations similar to what was proposed Rowe [1962], Matsuoka and Nakai [1977] and others. The resulting equation is a simple form:
\( \Delta \varepsilon^p = (\sin \psi) \Delta \gamma^p \) \hspace{1cm} (5)

Where: \( \psi \) is the dilation angle, which in turn is related to the constant volume friction angle \( \varphi_{cv} \) and the developed friction angle \( \varphi_d \) by:

\[
\sin \psi = (\sin \varphi_{cv} - \sin \varphi_d)
\]

This is a non-associated flow rule, since the direction of the plastic strain increment vector is not perpendicular to the yield loci, which are lines of constant stress ratio or friction angle.

2.4 UNDRAINED RESPONSE

The undrained behavior is captured by imposing the volumetric constraint caused by the fluid stiffness \( B_f \). The increment in pore-water pressure \( \Delta u \) is given by:

\[
\Delta u = \frac{B_f}{n} \Delta \varepsilon^f = \Delta \varepsilon_v
\]

Where: \( B_f \) is the fluid bulk modulus; \( n \) is the porosity of the soil skeleton; \( \Delta \varepsilon^f \) is the equivalent fluid volumetric strain; and \( \Delta \varepsilon_v \) is the volumetric strain of the soil skeleton

3. MONTE CARLO SIMULATION

The simulation by Monte Carlo can solve problems by generating suitable random numbers (or pseudo-random numbers) and assessing the dependent variable for a large number of possibilities. The Monte Carlo simulation (MCs) involves the definition of the variables that generate uncertainty and probability density function (pdf); determination of the value of the function using variable values randomly obtained considering the pdf; and repeating this procedure until a sufficient number of outputs to build the pdf of the function.

4. RESULTS AND DISCUSSION

To analyze the model under monotonic load, first the model is properly meshed. Then the monotonic load as incremental applied to model and displacements, strains, stresses and pore water pressure in different points of the model are computed. At the end, the desired result is analyzed. A random finite element analysis is used for reliability assessment of static liquefaction potential of loose sand under monotonic loading. The Monte Carlo simulation was used as reliability assessment approach. For Monte Carlo simulation in MATLAB, iterative numerical analysis is done. In this analysis, desired probability distribution for the random variables defined. Then by defining a loop, in any trial and error, value according to the variable distribution selected and introduces to the finite element model. If the soil is nonlinear elastic and/or elasto-plastic, the equivalent constitutive matrix \([D]\), is no longer constant, but varies with stress and/or strain. To be able to use the elasto-plastic constitutive models, to represent soil behavior in a finite element analysis, a solution strategy is required that can account for this changing material behavior. The modified Newton-Raphson method is used to consider the effect of changing material behavior in this research. In Figure (1), the used finite element model in this research is shown.
5. STOCHASTIC PARAMETERS

To account for uncertainties in the analysis of liquefaction, three input parameters have been considered as stochastic variables. The selected parameters are soil initial plastic shear modulus ($G_i^p$), unit weight ($\gamma$), and peak friction angle ($\phi_f$). These stochastic parameters are modeled using truncated normal Probability Density Functions (PDF). The distribution functions of the above mentioned stochastic parameters are as follows:

$$f_{G_i^p}(G) = \frac{1}{\sigma_{G_i^p}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{G - G_{i,\text{mean}}^p}{\sigma_{G_i^p}}\right)^2\right), \quad G_{i,\text{min}}^p \leq G_i^p \leq G_{i,\text{max}}^p \quad (8)$$

$$f_{\phi_f}(\phi_f) = \frac{1}{\sigma_{\phi_f}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\phi_f - \phi_{f,\text{mean}}}{\sigma_{\phi_f}}\right)^2\right), \quad \phi_{f,\text{min}} \leq \phi_f \leq \phi_{f,\text{max}} \quad (9)$$

$$f_{\gamma}(\gamma) = \frac{1}{\sigma_{\gamma}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\gamma - \gamma_{\text{mean}}}{\sigma_{\gamma}}\right)^2\right), \quad \gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}} \quad (10)$$

Where:

$$\begin{align*}
G_{i,\text{min}}^p &= G_{i,\text{mean}}^p \cdot 4\sigma_{G_i^p} \\
G_{i,\text{max}}^p &= G_{i,\text{mean}}^p + 4\sigma_{G_i^p} \\
\phi_{f,\text{min}} &= \phi_{f,\text{mean}} \cdot 4\sigma_{\phi_f} \\
\phi_{f,\text{max}} &= \phi_{f,\text{mean}} + 4\sigma_{\phi_f} \\
\gamma_{\text{min}} &= \gamma_{\text{mean}} \cdot 4\sigma_{\gamma} \\
\gamma_{\text{max}} &= \gamma_{\text{mean}} + 4\sigma_{\gamma}
\end{align*} \quad (11)$$
6. **EXAMPLE**

An example problem with arbitrary parameters for sandy soil is selected. The mean and standard deviation values of stochastic and deterministic parameters are given in Table (1) and (2) respectively. Figure (2) to (4) shows the probability density function of stochastic input parameters. The Monte Carlo simulation is selected as stochastic approach. For this purpose, 10,000 trials are used for the (MCs). Figures (5) to (10) shows PDF and CDF of mean total stress, pore water pressure and mean effective stress, respectively, obtained from analysis at point A.

### Table 1- Stochastic truncated normal parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit weight (kN/m³)</th>
<th>Initial plastic shear modulus (kPa)</th>
<th>Peak Friction angle (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>21</td>
<td>30000</td>
<td>33</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.6</td>
<td>5000</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 2- Model parameters used to simulate undrained behavior of loose sand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_B^e$</td>
<td>900</td>
<td>$K_G^e$</td>
<td>300</td>
</tr>
<tr>
<td>$\gamma'_w$ (kN / m³)</td>
<td>10</td>
<td>$P_s$ (kPa)</td>
<td>100</td>
</tr>
<tr>
<td>$B_f$ (kPa)</td>
<td>1e5</td>
<td>$m_e, n_e$</td>
<td>0.5</td>
</tr>
<tr>
<td>$n$</td>
<td>0.5</td>
<td>$n_p$</td>
<td>0.67</td>
</tr>
</tbody>
</table>

**Figure 2.** Probability density function of Peak friction angle

**Figure 3.** Probability density function of unit weight
Figure 4. Probability density function of initial plastic shear modulus

Figure 5. Probability density function of mean total stress

Figure 6. Cumulative density function of mean total stress

Figure 7. Probability density function of pore water pressure

Figure 8. Cumulative density function of pore water pressure
Figures (11) and (12) shows PDF and CDF of safety factor against liquefaction, obtained from the developed method at A, B and C points, respectively. It can be seen for this site that liquefaction has occurred first at or near the surface and worked its way downward and the probability of liquefaction decreases in depth.

9. CONCLUSION

Determination of liquefaction potential is a probabilistic problem due to the inherent uncertainties in estimation of soil parameters. In this research, a random finite element analysis is used for reliability assessment of static liquefaction potential of loose sand under monotonic loading. The Monte Carlo simulation was used for that purpose. The selected stochastic parameters were unit weight, peak friction angle and initial plastic shear modulus of soil which were modeled using truncated normal probability distribution function. The results showed that the probability distribution of liquefaction safety factor also has a near normal distribution. It can be seen for this site that liquefaction has occurred first at or near the surface and worked its way downward and the probability of liquefaction decreases in depth.
10. REFERENCES


