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An analytical reliability assessment of cohesive vertical cut stability

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Abstract
Slope stability analysis is a branch of geotechnical engineering that is highly amenable to probabilistic treatment. Probabilistic analysis of slope stability has received considerable attention in the literature, and has been used as an effective tool to evaluate uncertainty that is so prevalent in variables. In this research, the jointly distributed random variables method is used for probabilistic analysis and reliability assessment of the stability of cohesive vertical cut. The selected stochastic parameters are height, cohesion and unit weight, which are modeled using a truncated normal probability distribution function. The angle of slope relative to vertical is regarded as constant parameter. The results are compared with the Monte Carlo. Comparison of the results indicates the superior performance of the proposed approach for assessment of reliability.

KEYWORDS: Reliability, Jointly distributed random variables, Monte Carlo, slope stability.

1. INTRODUCTION

The problem of slope stability is a statically indeterminate problem. There are different methods of analysis available for engineers to assess the stability of slopes. It can be carried out by the Limit Equilibrium Method (LEM), the limit analysis method, the Finite Element Method (FEM) or the finite difference method. By far, most engineers still use the limit equilibrium method, with which they are more familiar. These methods are widely documented in geotechnical literature and use principles of static equilibrium to evaluate the balance of driving and resisting forces e.g., [1-4]. The factor of safety is defined as the ratio of resisting forces over driving forces, or, alternatively, as the shear strength divided by the calculated shear stresses. A factor of safety greater than one indicates a stable slope and a value less than one indicates impending failure. Therefore, these methods are restricted by the use of single valued parameters to describe the slope’s characteristics. However, the inherent uncertainties of the characteristics which affect slope stability dictate that the slope stability problem is of a probabilistic nature rather than being deterministic.

In general, the uncertainty in the stability of a slope is divided into three distinctive categories: soil parameter uncertainty, model uncertainty and human uncertainty[5]. Parameter uncertainty is the uncertainty in input parameters for analysis [6,7], model uncertainty is due to the limitation of theories and models used in performance prediction [8], while human uncertainty is due to human error[9]. In this research, parameter uncertainty is assessed.

2. SLOPE WITH PLANE SLIDING

When considering the local or micro-stability of a tunnel face, one often only considers the stability of soils with low or no cohesion in front of a slurry or air supported tunnel face. For the case of slurry supported face in cohesion less material Muller-Kirchenbauer [10] presented a solution for the minimal stagnation gradient of the slurry, required to ensure the stability of a small group of grains at the face. Combined with an estimate of the equivalent radius of the average flow channel between the grains, this solution can be used to estimate the minimal required yield strength of the slurry.

In cohesive soils one often disregards the local stability of the soil, as it is deemed secondary to the stability of the entire face. A problem could occur, however, in cases where a pressure gradient towards the face is present. This is easily conceivable in an earth-pressure balance shield with support pressures below the local hydrostatic pressure [11]. Unfortunately the general approach followed by Muller-Kirchenbauer does not lead to a straightforward solution in cohesive materials and a different approach is needed.
2.1 UPPER BOUND SOLUTION

The simple upper bound solution for the critical height of a vertical cut in a cohesive material is presented in many textbooks on soil mechanics, e.g. Verruijt [12]. For a vertical cut with height \( h \) assume a straight failure plane as sketched in figure 1. In this case the weight of the failure wedge is given by:

\[
W = \frac{1}{2} h^2 \gamma \tan \alpha
\]

and from a simple force vector plot it can be found that

\[
T = W \cos \alpha.
\]

This friction force \( T \) is also equal to

\[
T = \frac{h}{\cos \alpha} C
\]

Figure 1- Definition of forces on a triangular failure body

For a purely cohesive material with cohesion \( c \) and from the last two equations it can be easily derived that:

\[
h = \frac{4c}{\gamma \sin 2\alpha}
\]

for a given slope \( \alpha \). Failure will occur for that value of \( \alpha \) for which \( h \) is minimal, or \( \alpha = 45^\circ \). The critical height \( h_c \) is then found as

\[
h_c = \frac{4c}{\gamma}
\]

As an aside it can be noted that in 1927 Fellenius has found a lower upper bound solution using circular failure planes

\[
h_c = \frac{3.83c}{\gamma}
\]

Figure 1 shows a vertical cut in a purely cohesive soil. The critical sliding surface is inclined 45 degrees from the horizontal. The shear strength that resists sliding on this surface is \( c \). The shearing force that tends to cause sliding is \( \gamma H / 4 \). Therefore, the factor of safety is

\[
FS = \frac{4c}{\gamma h}
\]

3. STOCHASTIC PARAMETERS

To account for uncertainties in the analysis of the stability of vertical cut, 3 input parameters have been considered as stochastic variables. The selected parameters are height (\( h \)), cohesion (\( c \)), and unit weight (\( \gamma \)). These stochastic parameters are modeled using truncated normal probability distribution functions (PDF). The parameters related to geometry are regarded as constant parameters. The distribution functions of the above mentioned stochastic parameters are as follows:
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\[ f_h(h) = \frac{1}{\sigma_h \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{h - h_{\text{mean}}}{\sigma_h} \right)^2 \right) \quad h_{\text{min}} \leq h \leq h_{\text{max}} \]

\[ f_c(c) = \frac{1}{\sigma_c \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{c - c_{\text{mean}}}{\sigma_c} \right)^2 \right) \quad c_{\text{min}} \leq c \leq c_{\text{max}} \]

\[ f_\gamma(\gamma) = \frac{1}{\sigma_\gamma \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{\gamma - \gamma_{\text{mean}}}{\sigma_\gamma} \right)^2 \right) \quad \gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}} \]

Where:

\[
\begin{align*}
h_{\text{min}} &= h_{\text{mean}} - 3\sigma_h \\
h_{\text{max}} &= h_{\text{mean}} + 3\sigma_h \\
c_{\text{min}} &= c_{\text{mean}} - 3\sigma_c \\
c_{\text{max}} &= c_{\text{mean}} + 3\sigma_c \\
\gamma_{\text{min}} &= \gamma_{\text{mean}} - 3\sigma_\gamma \\
\gamma_{\text{max}} &= \gamma_{\text{mean}} + 3\sigma_\gamma
\end{align*}
\]

By considering the stochastic variables within the range of their mean, plus or minus 3 times standard deviation, 99.8\% of the area beneath the normal density curve is covered. Thus, area correction will not be necessary. It should be noted that for choosing initial data, the following conditions must be observed for height, cohesion and unit weight:

\[
\begin{align*}
h_{\text{mean}} - 3\sigma_h &> 0 \\
c_{\text{mean}} - 3\sigma_c &> 0 \\
\gamma_{\text{mean}} - 3\sigma_\gamma &> 0
\end{align*}
\]

4. **JOINTLY DISTRIBUTED RANDOM VARIABLES METHOD**

Jointly Distributed Random Variables method (JDRV method) is an analytical probabilistic method. In this method, density functions of input variables are expressed mathematically and joined together by statistical relations. The available statistical and probabilistic relations between parameters are given in this section [13,14].

If X is a random variable with the probability density of \( f_X(x) \), and Y is a function of X in the form \( Y = g(x) \), the probability density of Y can be determined as:

\[ f_Y(y) = f_X \left( g^{-1}(y) \right) \left| \frac{dg^{-1}}{dy} \right| \]

If X and Y are two independent random variables with the probability densities \( f_X(x) \) and \( f_Y(y) \), and \( Z = X + Y \), the probability density of Z will be:

\[ f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x) dx \quad (-\infty < z < +\infty) \]

If X and Y are two random variables with the probability densities \( f_X(x) \) and \( f_Y(y) \), and \( Z = Y/X \), the probability density of Z will be:

\[ f_{Y/X}(z) = \int_{-\infty}^{\infty} |x| f_X(x) f_Y(zx) dx \quad (-\infty < z < +\infty) \]

If X and Y are two random variables with the probability densities \( f_X(x) \) and \( f_Y(y) \), and \( Z = Y.X \), the probability density of Z will be:

\[ f_{XY}(z) = \int_{-\infty}^{\infty} 1/|x| f_X(x) f_Y(z/x) dx \]

5. PROBABILISTIC ASSESSMENT OF VERTICAL CUT STABILITY

In this research, the terms of the safety factor equation (Eq. (7)) are grouped together in the following form (Eq. (17)), and the probability distribution equation of each group is derived separately using Eqs. (18) to (22). Derivations of these equations are given below:

\begin{align*}
\{k_1 = 4c \\
    k_2 = \gamma \\
    k_3 = h \\
    k_4 = \gamma h \\
    k_5 = 4c / \gamma h
\end{align*}

On the other hand:

\begin{align*}
f_{k_1}(k_1) &= \frac{1}{4\sigma_c \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{k_1 - 4c_{\text{mean}}}{4\sigma_c} \right)^2 \right) \quad 4c_{\text{min}} \leq k_1 \leq 4c_{\text{max}} \\
f_{k_2}(k_2) &= \frac{1}{\sigma_\gamma \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{k_2 - \gamma_{\text{mean}}}{\sigma_\gamma} \right)^2 \right) \quad \gamma_{\text{min}} \leq k_2 \leq \gamma_{\text{max}} \\
f_{k_3}(k_3) &= \frac{1}{\sigma_h \sqrt{2\pi}} \exp \left( -0.5 \left( \frac{k_3 - h_{\text{mean}}}{\sigma_h} \right)^2 \right) \quad h_{\text{min}} \leq k_3 \leq h_{\text{max}} \\
f_{k_4}(k_4) &= \frac{\beta}{k_2} \int_{k_2}^{k_4} f_{k_2}(k_2) f_{k_1}(k_4 - k_2) dk_2
\end{align*}

Where:

\begin{align*}
\alpha &= \max \left[ k_{2\text{min}}, \frac{k_4}{k_{3\text{max}}} \right] \\
\beta &= \min \left[ k_{2\text{max}}, \frac{k_4}{k_{3\text{min}}} \right]
\end{align*}

\begin{align*}
f_{k_4}(k_4) &= \int_{\alpha}^{\beta} f_{k_4}(k_4) f_{k_5}(k_5 - k_4) dk_5
\end{align*}

Where:

\begin{align*}
\alpha &= \max \left[ k_{4\text{min}}, \frac{k_{1\text{min}}}{k_{5\text{max}}} \right] \\
\beta &= \min \left[ k_{4\text{max}}, \frac{k_{1\text{max}}}{k_{5\text{min}}} \right]
\end{align*}

Using the above mathematical functions for \(k_1\) to \(k_5\) and \(f_{K_1}(k_1)\) to \(f_{K_5}(k_5)\), a computer program was developed (coded in Matlab) to determine the probability density distribution curve for the factor of safety for infinite slopes. In addition, for comparison, determination of the safety factor for a vertical cut, using Monte Carlo simulation, was also coded in the same computer program. To illustrate the capabilities of this method, an example with arbitrary data is given in the section 7.
6. MONTE CARLO METHOD

The Monte Carlo (MC) method is a computational algorithm that relies on repeated random sampling to address risk and uncertainty in quantitative analysis and decision making. This method provides a range of possible outcomes and the probabilities that they will occur for any choice of action. The MC method involves building models of possible results by substituting a range of values (a probability distribution) for any variable with inherent uncertainty. It then calculates results over and over, each time using a different set of random values from the probability functions. Depending on the extent of uncertainty and the ranges specified for the variables, this method could involve a significant number of simulation runs to produce distributions of possible outcome values.

During a Monte Carlo method, values are sampled at random from the input probability distributions. Each set of samples and the resulting outcome from that sample are recorded. This method provides a probability distribution of possible outcomes and, hence, gives a much more comprehensive view of what may happen.

7. EXAMPLE

To demonstrate the efficiency and accuracy of the proposed method in determining the probability density distribution curve of the safety factor, vertical cut is considered. The mean and variance of the stochastic parameters are selected as $c_{\text{mean}} = 20 \text{kPa}$, $\sigma_c = 3 \text{kPa}$ for cohesion of the soil, $h_{\text{mean}} = 7 \text{m}$, $\sigma_h = 0.2$ for height of vertical cut, and $\gamma_{\text{mean}} = 15 \text{kN/m}^3$, $\sigma_\gamma = 0.5 \text{kN/m}^3$ for unit weight of soil. Angle of the vertical cut is considered as deterministic (non-stochastic) parameters. In order to compare the results of the JDRV method with those of the MC method, the final probability density distribution curves for the factor of safety are determined using the same data and this method. For this purpose 1000,000 generation points are used for the MC method.
Figure 2- Probability density function of input model parameter.

Figure 3- Comparison of probability distribution functions of the factor of safety by MC method and JDRV method.

Figure 4- Probability of failure in the example problem.
7. CONCLUSION

Slope stability analysis is a probabilistic problem, due to the inherent uncertainties in geotechnical parameters, model performance, and human uncertainty. A number of attempts have been made for probabilistic assessment of this type of geotechnical problems. In this paper, the JDRV method and MC method were used to assess the reliability of vertical cut stability problems, based on the uncertainty in geotechnical properties. The selected stochastic parameters were height, cohesion and unit weight. The results showed that the probability distribution of the safety factor also has a nearly normal distribution and compares favorably with the output of other method of analysis, the MC method. The results also indicated that the JDRV method was able to capture the expected probability distribution of the safety factor of a vertical cut correctly. This method can be used as a suitable method in assessment of the reliability of vertical cuts. The JDRV method has a number of advantages over other methods:

(i) It is an exact method, and can be used for stochastic parameters with any distribution curve (such as normal, exponential, gamma, uniform).

(ii) The computational time of this method is significantly less than the MC method, which requires a significant number of simulation runs.

9. REFERENCES