PHN10106101213

Prediction of strain localization in granular soils

Armaghan Badie¹, Ali Lashkari²
1- M.Sc. student in Geomechanics, Department of Civil & Environmental Engineering, Shiraz University of Technology, Shiraz, Iran
2- Assistant professor, Department of Civil & Environmental Engineering, Shiraz University of Technology, Shiraz, Iran

Corresponding Author’s E-mail (lashkari@sutech.ac.ir)

Abstract

Strain localization is a well-known phenomenon in geomaterials. In this process, plastic deformation is observed to concentrate in narrow zones called shear bands. The accurate numerical simulation of the slope stability problems, the bearing capacity of shallow and deep foundations, and the active and passive stresses acting on retaining walls depend strongly on formation of shear bands in granular soils. In this study, an advanced plasticity model formulated within the bounding surface plasticity family and critical state soil mechanics is applied for prediction of strain localization in granular soils. It is shown that realistic predictions are obtained for states at which strain localization can take place.

Keywords: Sand, Bounding surface plasticity, Dilatancy, Hardening.

1. INTRODUCTION

Under loading conditions found in many geotechnical structures, it is common to observe failure in zone of intense localized strain called shear bands. This event which is a fundamental phenomenon in granular materials has been widely investigated during recent decades by both experimental and theoretical approaches [1-6]. As for soils and granular materials, extensive experimental results with respect to formation of shear banding have been reported in the literature [1-4]. Granular materials deform in various ways beneath the engineering structures. For small strains, deformation is nearly uniform. However, for large strains, deformation may localized in one or more narrow regions of soil mass. These regions, technically called shear bands, separate almost rigid blocks of granular media. Researchers have recently carried out systematic studies to analyze and describe the occurrence and patterns of shear bands. The theoretical studies have used bifurcation theory to predict and describe the occurrence of shear bands [5,6]. Prediction of strain localization depends strongly on the constitutive models employed to describe the mechanical behavior of soil. Thus, shear band analysis is closely coupled with the constitutive relations. Herein, in order to investigate the stress and density dependent behavior of granular soils, a unified description of the interaction between the mean stress and the current density is necessary. As a result, a state-dependent bounding surface plasticity model is used to describe the three-dimensional stress-strain behavior of granular soils. Predictions for strain localization are compared with data of compression mode of triaxial. It is shown that a reasonable agreements exists between the model predictions and experimental data.

2. CONSTITUTIVE MODEL IN MULTIAXIAL STRESS AND STRAIN SPACES

The elastoplastic model selected here is essentially the constitutive model proposed by Manzari & Dafalias [7]. In the following lines, the model formulation is presented:

The rate of strain tensor can be decomposed into elastic and plastic parts indicated, respectively, by “e” and “p” superscripts:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$$  (1)

For the elastic branch of behavior, the isotropic hypo-elasticity is defied by:
\[
\dot{\mathbf{s}} = 2G \dot{\mathbf{e}}; \quad \dot{\mathbf{p}} = K \dot{\mathbf{e}}^p
\]  

where, \(p(=1/3\sigma_{kk})\) is mean principal effective stress, and \(\mathbf{s} = \mathbf{\sigma} - 1/3\sigma_{kk} \mathbf{I}\) is deviator stress tensor in which \(\mathbf{I}\) is the second order identity tensor. In a similar fashion, \(\mathbf{e}^p\) and \(\mathbf{e}_v\) are, respectively, deviator strain tensor and volumetric strains. \(G\) and \(K\) are elastic shear and bulk moduli, respectively, given by:

\[
G = G_0 \frac{(2.973 - e)^3}{1 + e} \sqrt{\frac{p}{p_{\text{ref}}}}; \quad K = \frac{2G}{3} \left(1 + \frac{v}{1 - 2v}\right)
\]

In which, \(G_0\) and \(v\) are model parameters and \(p_{\text{ref}} (=101\ \text{kPa})\) is a reference normalizing pressure. \(e\) is the current value of void ratio.

In this model, yield surface is in the shape of a circular cone whose apex is on the origin of stress space:

\[
f(\mathbf{\sigma}, \mathbf{a}) = [(\mathbf{s} - \mathbf{p}\mathbf{a}) : (\mathbf{s} - \mathbf{p}\mathbf{a})]^{1/2} - \sqrt{2/3} mp = 0
\]  

In Eq. (4), \(\mathbf{a}\) is back-stress ratio tensor that defines the location of yield surface center in deviator plane and \(m(=0.01)\) indicates the yield function size. When the current stress state reaches the yield surface and attempts to move beyond, plastic strains begin to generate. At this situation, normal to yield surface is:

\[
L = \frac{\partial f(\mathbf{\sigma}, \mathbf{a})}{\partial \mathbf{a}} = \mathbf{n} - \frac{1}{3} N \mathbf{I}
\]

where \(\mathbf{n} = \frac{s - p\mathbf{a}}{\sqrt{2/3} mp}\) and \(N = \frac{1}{p}\).

A non-associated flow rule is used in the model. The following constitutive equation calculates plastic strain rate:

\[
\dot{\mathbf{e}}^p = \mathbf{\lambda} \mathbf{R} = \mathbf{\lambda} \left[\mathbf{n} + \frac{1}{3} \mathbf{D} \mathbf{I}\right]
\]

In above equation, \(\mathbf{\lambda}\) is loading index defining the magnitude of plastic strain rate, and \(\mathbf{D}\) is dilatancy.

Consistency condition dictates the following mathematical form for loading index:

\[
\mathbf{\lambda} = \frac{1}{K_p} \mathbf{L} : \mathbf{\sigma} = \frac{2G \mathbf{n} : \dot{\mathbf{e}} - KN \dot{\mathbf{e}}_v}{K_p + 2G - KND} \geq 0
\]  

where, \(K_p\) is plastic hardening modulus.

Plastic hardening modulus and dilatancy are defined through:

\[
K_p = h_0 (1 - c_h e) G \frac{a^b(\theta) - \mathbf{a}}{\mathbf{a} : \mathbf{n}}; \quad d = A (a^d(\theta) - \mathbf{a}) : \mathbf{n}
\]

where, \(h_0, c_h,\) and \(A\) are model parameters. \(\theta\) is Lode angle. \(a^b\) and \(a^d\) are two second order tensors defined by:

\[
a^b(\theta) = \sqrt{2/3} [g(\theta, c) M_c \exp(-n^b \psi) - m] \mathbf{n}; \quad a^d(\theta) = \sqrt{2/3} [g(\theta, c) M_c \exp(n^d \psi) - m] \mathbf{n}
\]

where, \(n^b\) and \(n^d\) are model parameters. \(c = M_s/M_c\) in which \(M_c\) and \(M_s\) are, respectively, slopes of critical state line under the compression and extension modes of triaxial. \(g(\theta, c)\) is a function whose major role is considering the Lode angle effects on \(a^b\) and \(a^d\). finally, \(\psi\) is state parameter:

\[
\psi = e - e_c = e - \left(e_0 - \frac{\mathbf{p}}{p_{\text{ref}}} \right)^\xi
\]  

\[\xi\]
In Eq. (10), \( e_0, \lambda, \) and \( \xi \) are model parameters.

3. **Critical condition for strain localization**

In modern geomechanics, strain localization is considered as an example of lack of elastoplastic uniqueness. Herein, the approach of Lade [3] for localization analysis is followed. Considering Eqs. (6), Eq. (1) can be re-written as:

\[
\Delta \varepsilon = C^e \Delta \sigma + \frac{1}{K_p} (L : \Delta \sigma) R
\]

Eq. (11) can be expanded into the following form:

\[
\begin{cases}
\Delta \varepsilon_{11} = 0 \\
\Delta \varepsilon_{22} = 0 \\
\Delta \varepsilon_{33} \neq 0 \\
\Delta \varepsilon_{23} = 0 \\
\Delta \varepsilon_{13} \neq 0 \\
\Delta \varepsilon_{12} = 0
\end{cases}
\begin{pmatrix}
C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1113} & C_{1112} \\
C_{2211} & C_{2222} & C_{2233} & C_{2223} & C_{2213} & C_{2212} \\
C_{3311} & C_{3322} & C_{3333} & C_{3323} & C_{3313} & C_{3312} \\
C_{2311} & C_{2322} & C_{2333} & C_{2323} & C_{2313} & C_{2312} \\
C_{1311} & C_{1322} & C_{1333} & C_{1323} & C_{1313} & C_{1312} \\
C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1213} & C_{1212}
\end{pmatrix}
\begin{pmatrix}
\Delta \sigma_{11} \\
\Delta \sigma_{22} \\
\Delta \sigma_{33} \\
\Delta \sigma_{23} \\
\Delta \sigma_{13} \\
\Delta \sigma_{12}
\end{pmatrix}
= 0
\]

Eq. (12) necessitates:

\[
C_{1111}C_{2222} - C_{1122}C_{2211} = 0
\]

In which

\[
\begin{align*}
C_{1111} &= \frac{1}{2(1+\nu)}G + \frac{1}{K_p}L_{11}R_{11} \\
C_{2222} &= \frac{1}{2(1+\nu)}G + \frac{1}{K_p}L_{22}R_{22} \\
C_{1122} &= -\nu + \frac{1}{K_p}L_{11}R_{22} \\
C_{2211} &= -\nu + \frac{1}{K_p}L_{22}R_{11}
\end{align*}
\]

Substitution of Eqs. (15) into Eq. (14) gives the value of plastic hardening modulus for which localization may take place:

\[
K_p = \frac{2}{\nu - 1}G \left[ \nu(L_{11}R_{22} + L_{22}R_{11}) + (L_{11}R_{11} + L_{22}R_{22}) \right]
\]

3. **Critical condition for strain localization in triaxial space**

Eq. (16) can be further extended with respect to the compression mode of triaxial. To this aim it should be expressed that, shear banding takes place in a critical plane that makes angle \( \delta \) with horizontal plane. Considering this configuration, \( L_{11} \) and \( R_{11} \) can be calculated based on corresponding principal values:

\[
L_{11} = \frac{1}{2}(L_1 + L_{III}) - \frac{1}{2}(L_1 - L_{III}) \cos(2\delta) \\
R_{11} = \frac{1}{2}(R_1 + R_{III}) - \frac{1}{2}(R_1 - R_{III}) \cos(2\delta)
\]
In Eq. (17), \( L_1 = \partial \sigma / \partial \sigma_1 = n_1 - 1/3N \) and \( L_{III} = \partial \sigma / \partial \sigma_{III} = n_{III} - 1/3N \) in which, \( \sigma_1 \) and \( \sigma_{III} \) are major (i.e., vertical) and minor (i.e., horizontal) principal stresses in triaxial space. \( n_1 \) and \( n_{III} \) are, respectively, \( \partial \sigma / \partial \sigma_I \) and \( \partial \sigma / \partial \sigma_{III} \). In a similar manner and considering Eq. (6), one has \( R_1 = n_1 + 1/3D \) and \( R_{III} = n_{III} + 1/3D \). Finally, one has \( L_{II} = \partial \sigma / \partial \sigma_{II} = n_{II} - 1/3N \) and \( R_{II} = n_{II} + 1/3D \). Substitution of Eq. (17) in Eq. (16) gives the following relationship for \( \delta \) at strain localization:

\[
\sin \delta_c = \pm \frac{1}{2} \sqrt{ 2 - \left( \frac{2vL_{II} + L_1 + L_{III}}{L_1 - L_{III}} \right) + \left( \frac{2vR_{II} + R_1 + R_{III}}{R_1 - R_{III}} \right) } \tag{18}
\]

For practical application, implementation of Eq. (18) into Eq. (16) gives the critical value of plastic hardening modulus at which strain localization may happen. The constitutive framework introduced in section 2 is a critical state compatible bounding surface plasticity model. In this formulation, plastic hardening modulus is infinity when a new loading starts; however, it continuously decreases as stress state approaches towards the bounding surface. In this procedure, once the plastic hardening modulus becomes identical with the critical value obtained from the implementation of Eq. (18) in Eq. (16), strain localization in form of shear banding becomes possible. Prior to this moment, shear banding is theoretically impossible.

### 4. Evaluation of the Model in Prediction of Strain Localization

Desrues & Viggiani [4] studied the occurrence of shear banding in dense Hostun RF sand under the compression mode of triaxial. Hostun RF is a uniformly graded silica sand with \( d_{90}=0.32 \) mm, \( Cu=1.7 \) and \( G_s=2.65 \). For this sand, the maximum and minimum dry densities are 15.99 and 13.24 kN/m\(^3\), respectively. All triaxial tests were performed under drained condition for confining pressure values of 100, 200, 400, and 800 kPa and the initial relative density of samples was around 95% \( (\epsilon_r=0.67) \). For four samples of Hostun RF sand described above, the model predictions are depicted against experimental data in Fig. 1. Parameters used in all simulations are given in Table 1. In predictions, states for which localization is not possible are shown in red color. However, states for which localization is permitted are drawn in blue. For each test, the first state at which plastic hardening modulus is identical to the critical one (see Eqs. (17) and (18)) are indicated by circle. A reasonable correspondence can be observed between those states indicated by circles and the actual localized states. It should be noted that in actual localized states, an abrupt drop in the mobilized friction and angle is observed. Further, at such states, dilation is suddenly stopped due to the so-called elastic unloading in the post-bifurcation regime of behavior. In the current analysis, simulation of the post-bifurcation analysis is not possible. For this aim, one can invoke to advanced methods such as non-local approaches. Also in Fig. 2, the predicted shear band inclination angle and axial strain at localization point are depicted with the corresponding observed values in experiments.

Comparing the model predictions and data in Figs. 1 and 2, the following points may be indicated:

- Localization always occurred in dilative branch of the behavior.
- The mobilized friction angle immediately prior to localization is larger than the critical state value of friction angle.
- For larger values of initial mean principal effective stress values (i.e., confining stress), localization may be occurred at lower values of mobilized friction angle.
- The increase in initial value of mean principal effective stress values (i.e., confining stress) leads to the increase in axial strain at which localization occurred.

### Table 1- The model parameters used in simulations

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Critical state</th>
<th>Dilatancy</th>
<th>Plastic hardening modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>G_s=80</td>
<td>M=1.3</td>
<td>A=0.85</td>
<td>h_b=9.7</td>
</tr>
<tr>
<td>( v=0.12 )</td>
<td>c=0.81</td>
<td>n_b=3.97</td>
<td>( c_b=0.8 )</td>
</tr>
<tr>
<td></td>
<td>e_0=0.955</td>
<td></td>
<td>( n^b=2.85 )</td>
</tr>
<tr>
<td></td>
<td>( \lambda=0.094 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \xi=0.30 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1. Simulations vs. experiments of four drained tests on Hostun RF sand: (a) mobilized friction angle against axial strain, and (b) volumetric strain against axial strain (data from [4])

5. CONCLUSIONS

In this study, the influence of stress level on localization properties was studied. To this aim, an advanced critical state compatible bounding surface plasticity was employed to describe soil behavior. It was shown that the onset of localization in form of shear bands is significantly affected by confining pressure level and density. The localization is retarded by increasing mean stress and by decreasing sample density. For a given mean stress, shear bands are steeper in dense specimens than in loose specimens. However, for both dense and loose sand, increasing mean stress contribute to decreasing shear band inclination angle.
Figure 2. Simulations vs. experiments of four drained tests on Hostun RF sand: (a) shear band inclination angle against initial value of mean principal effective stress (i.e., confining stress), and (b) axial strain at localization against initial value of mean principal effective stress (i.e., confining stress) (data from [4]).

6. REFERENCES