A constitutive model for unsaturated soil-structure interfaces

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Abstract

In geotechnical engineering structures, there may be a thin layer of soil between soil and structure through which, load transferring occurs. This layer is called soil-structure interface and the mechanical response of geostuctures depends strongly on the behavior of this layer. If the soil mass is in unsaturated state, then the interface is technically called unsaturated interface. In this paper, a constitutive model for unsaturated soil-structure interfaces is introduced. The soil-water characteristic curve and the concept of modified effective stress are adopted to create an appropriate hydro-mechanical coupling. The model predictive capacity is evaluated by comparing its predictions with the existing experimental data.

Keywords: Constitutive model, Matrice suction, Critical state, Unsaturated interface.

1. INTRODUCTION

To understand the behavior of most geotechnical constructions such as deep foundations, retaining walls, soil-nailing, off-shore, and underground structures, knowledge on the mechanical behavior of their interfaces with soil is essential. Early experimental researches into soil-structure interfaces have performed by means of direct shear [1-2], ring shear [3], simple shear [4-5] devices and particle image velocimetry [6] technique. According to these studies, behavior of soil-structure interfaces depends on various factors including stress path, density, normal stress, normal stiffness, structure roughness, mineralogy and degree of saturation.

Modeling of soil-structure interfaces is a young area in geotechnical engineering. For instance Clough and Duncan [8] developed an interface model based on the hyperbolic elasticity. Ghionna and Mortara [1], Shahrou and Rezaie [9], Fakharian and Evgin [10] among others, introduced constitutive models for soil-structure interfaces in the framework of elastoplasticity theory. De Gennaro and Frank [11] developed a constitutive model which can simulate normal dilatancy in soil-structure interface behavior. Recently, Lashkari [12-15] using the framework of bounding surface plasticity and critical state soil mechanics, proposed a constitutive model to simulate the behavior of sand-structure interfaces. The most important advantage of this model is to predict the mechanical response of interfaces in wide ranges of densities and applied normal stresses, using a unique set of parameters. Recently, Lashkari [16], used this model to predict the shaft resistance of non-displacement piles in sand.

In practical problems, often, we deal with partially saturated soils, whereas, most researches on soil-structure interfaces have done considering fully saturated or dry states. The first attempts to understand the behavior of unsaturated interfaces are those of Miller & Hamid [17], Hamid and Miller [18] and Khoury et al.[19]. By using the framework of two independent stress variables, Hamid and Miller [20] introduced the first constitutive model for unsaturated soil-structure interfaces. The biggest weakness of this model is that, for wide ranges of states and suction, it needs several sets of parameters to predict the behavior of interfaces. Recently, based on the critical state soil mechanics and two independent stress variable concept, Lashkari [22] introduced an extension to his previous interface model (e.g., [12-16]) for simulation of unsaturated interfaces. This model is capable of predicting the behavior of unsaturated sand-structure interfaces for different ranges of density, normal stresses, and suction values by using a unique set of parameters. Recent studies revealed that the framework of two independent stress variables responds to describe the behavior of unsaturated soils, only when, one of the variables is kept constant [23]. Furthermore, this framework is unable to appropriately simulate the hydro-mechanical coupling. Regarding these issues, Houslsby [24] demonstrated that two sets of stress-strain (like) variables are necessary to describe unsaturated soil behavior: the modified effective stress and suction as stress variables and the strain of solid skeleton and...
degree of saturation as strain variables. Houlsby's explanation is powerfully able to describe the behavior of unsaturated soils and it can simulate the hydro-mechanical coupling phenomena observed in unsaturated soils. Based on the latter approach, this paper presents a new form of the Lashkari [12-15] interface model to simulate the behavior of unsaturated interfaces based on the recent generation of effective stress for unsaturated geomaterials.

2. Stress Field Vector

The modified effective stress vector is defined by:

\[
\sigma^* = \begin{bmatrix} \sigma_n^* \\ \tau \end{bmatrix} = \begin{bmatrix} \sigma_{net} + \chi s \\ \tau \end{bmatrix}
\]  

(1)

where, \( \sigma_n^* \) and \( \tau \) are respectively, modified effective normal and tangential components of the stress vector, with respect to the interface plane. In Eq. (1), \( \sigma_{net} \) is the net normal stress, \( s \) is suction and \( \chi \) is the modified effective stress coefficient which in this research, is calculated through the following equation [25]:

\[
\chi = \frac{S_r - S_{r0}}{1 - S_{r0}}
\]  

(2)

where, \( S_r \) is the current degree of saturation and \( S_{r0} \) is the degree of saturation at residual state which can be obtained from the soil-water characteristic curve.

3. Mechanical Part of the Formulation

Corresponding to the stress field of Eq. (1), the relative displacement vector is defined as:

\[
\Delta = \begin{bmatrix} u \\ v \end{bmatrix}
\]  

(3)

where, \( v \) and \( u \) are, respectively, normal and tangential displacements, regarding to the interface plane.

The relative displacement rate vector is divided into elastic and plastic parts as follow:

\[
\dot{\Delta} = \dot{\Delta}^e + \dot{\Delta}^p = \begin{bmatrix} \dot{u}^e \\ \dot{v}^e \end{bmatrix} + \begin{bmatrix} \dot{u}^p \\ \dot{v}^p \end{bmatrix}
\]  

(4)

where, superscripts \( e \) and \( p \) indicate the elastic and plastic parts of the displacement vector, respectively.

To relate the rates of the modified effective stress and displacement vectors, the following constitutive law is adopted:

\[
\sigma^* = \frac{1}{t} D^{\text{op}} \Delta
\]  

(5)

where, \( t \) is the thickness of interface layer that is about 5-20 times of mean grain diameter \( (D_{50}) \) (e.g., [2, 16]). \( D^{\text{op}} \) is the elastoplastic stiffness matrix which is calculated by:

\[
D^{\text{op}} = D^{e} - \frac{D^{e} R \tau T D^{e}}{K_p + n T D^{p} R}
\]  

(6)

in Eq. (6), \( D^{e} \) is the elastic stiffness matrix and is defined by:

\[
D^{e} = \begin{bmatrix} K_n^e & 0 \\ 0 & K_t^e \end{bmatrix} - \begin{bmatrix} K_n^{e0} \sqrt{\sigma_n^*/P_{ref}} & 0 \\ 0 & K_t^{e0} \sqrt{\sigma_n^*/P_{ref}} \end{bmatrix}
\]  

(7)

In above equation, \( K_n^{e0} \) and \( K_t^{e0} \) are model parameters and \( P_{ref} \) is a reference pressure which commonly is taken as the atmospheric pressure (i.e., 101 kPa). By using elastic stiffness matrix, the rate of modified effective stress vector has been related to the rate of elastic displacement vector through the following constitutive equation:

\[
\sigma^* = \frac{1}{t} D^{e} \Delta^e
\]  

(8)

The following wedge-shaped yield function is defined in model:

\[
f = \tau - \eta \sigma_n^* = 0
\]  

(9)

where, \( \eta \) is stress ratio ( \( \eta = \frac{\tau}{\sigma_n^*} \) ).

The yield direction vector, \( \mathbf{n} \), is normal to the yield function as follows:

\[
\mathbf{n} = \begin{bmatrix} n_x \\ n_y \end{bmatrix}
\]  

(9a)
In Eq. (6), $K_p$ is the plastic hardening modulus and is defined by:

$$K_p = h_0 K_f \left( \frac{\exp(-n^b \psi)}{\eta} - 1 \right)$$

(11)

where, $h_0$ and $n^b$ are model parameters, $M$ is stress ratio at critical state and $\psi$ is the state parameter which will be introduced later.

In this model, the rate of the plastic displacement is calculated by using the following equation:

$$\dot{\mathbf{A}}^p = \left( \frac{1}{K_p} \mathbf{n}^T \mathbf{r} \right) \mathbf{R}$$

(12)

In Eq. (12), $\mathbf{R}$ is a vector defining direction of the plastic displacement rate which:

$$\mathbf{R} = \begin{bmatrix} R_n \\ R_t \end{bmatrix} = \begin{bmatrix} d \\ 1 \end{bmatrix}$$

(13)

where, the term $d$ in Eq. (13) indicates dilatancy function and is introduced through the following equation:

$$d = A \left( \exp(n^d \psi) - \frac{\eta}{M} \right)$$

(14)

in which, $n^d$ is a model parameter and $\psi$ is the state parameter of Been and Jefferies [26]:

$$\psi = e - e_c$$

(15)

where, $e$ and $e_c$ are, respectively, the current and critical state void ratios. To obtain a unique critical state line for wide ranges of normalized states, the approach of Gallipoli et al. [27] is adopted through which, the critical state line is calculated by:

$$e_c = e_0 - \lambda \ln \left( \frac{\sigma'_n/\sigma_{\text{ref}}}{1 + a \exp(b \xi - 1)} \right)$$

(16)

In Eq. (16), $e_0$, $\lambda$, $a$ and $b$ are model parameters related to the critical state line and $\xi$ is the bonding variable which has been introduced by Gallipoli et al. [27]:

$$\xi = f(s)(1 - S_v)$$

(17)

In the above equation, $f(s)$ is a function of suction which varies between 1 and 1.5, and in the present formulation is estimated through the following equation:

$$f(s) = 1 + \frac{0.5 s^{0.75}}{125 + s^{0.75}}$$

(18)

Eventually, in Eq. (14), the coefficient $A$ is introduced as (e.g., Lashkari [13-16, 22]):

$$A = A_0 \sqrt{\frac{\sigma_{\text{ref}}}{\sigma_n}} + A_1 - A_0 \sqrt{\frac{\sigma_{\text{ref}}}{\sigma_n}} \frac{\eta}{M} \exp(n^b \psi)$$

(19)

In Eq. (19), $A_0$ and $A_1$ are model parameters.

4. Hydraulic Part of the Formulation

The correlation between the degree of saturation and suction is reflected by soil-water characteristic curve (SWCC). Among various suggestions in the literature, the following relationship proposed by Van Genuchten [28] is adopted in this paper:

$$S_v = S_{\text{at}} + (1 - S_{\text{at}}) \left[ 1 + \left( \frac{s e \Omega}{a_w P_{\text{ref}}} \right)^{n_v} \right]^{\frac{1}{n_v} - 1}$$

(20)

where, $a_v$, $\Omega$ and $n_v$ are parameters related to the SWCC curve and $S_{\text{at}}$ is the degree of saturation at residual state. By selecting different sets of parameters, the wetting and drying branches of the SWCC curve can be described. The term $e^{\Omega}$ reflects the effect of the soil density on the SWCC curve.
5. **The Model Verification**

In this section, the model predictive capacity is evaluated by comparing with two sets of experimental data. The first set of data are those of Hamid and Miller [18], which were conducted using stainless steel plate adjacent to the Minco silt. The steel plate was 25.5 mm thick and 102 mm in diameter with rough surface geometry. Properties of the Minco silt are presented in Table 1. Initial void ratio and degree of saturation are estimated 0.466 and 0.86, respectively. The SWCC curve related to this set of data is depicted in Fig. 1(a) and the model parameters are given in Table 2. In all simulations, the interface thickness is taken 1 mm.

### Table 1 - The physical properties of Minco silt [18]

<table>
<thead>
<tr>
<th>Properties</th>
<th>Mean diameter, $D_{50}$ (mm)</th>
<th>Liquid limit, LL (%)</th>
<th>Plasticity index, PI (%)</th>
<th>Minimum void ratio, $e_{min}$</th>
<th>Fines content (%)</th>
<th>Specific gravity of grain, $G_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.04</td>
<td>28</td>
<td>8</td>
<td>0.485</td>
<td>73</td>
<td>2.67</td>
</tr>
</tbody>
</table>

### Table 2 - The model parameters for rough unsaturated Minco-silt-steel interface

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$K^0_{s0}$</th>
<th>$K^0_{s0}$</th>
<th>$\sigma_0$</th>
<th>$\lambda$</th>
<th>$M$</th>
<th>$a$</th>
<th>$b$</th>
<th>$m$</th>
<th>$n$</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$h_0$</th>
<th>$S_{so}$</th>
<th>$\Omega$</th>
<th>$a_s$</th>
<th>$n_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>300</td>
<td>310</td>
<td>0.12</td>
<td>0.15</td>
<td>0.54</td>
<td>0.5</td>
<td>14</td>
<td>0.3</td>
<td>2</td>
<td>0.65</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.12</td>
<td>0.09</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Comparisons between the model predictions and experimental data of Hamid and Miller [18], for different values of suctions and net normal stresses, are shown in Figs. 2-4. In Fig. 2, the effect of suction on the dilative and shear strength behavior of unsaturated interfaces is studied. As it can be observed, by increasing suction from 50 to 100 kPa, at the constant net normal stress 140 kPa, the maximum shear strength and dilative behavior are significantly increased, but the post peak shear strength is nearly remained unchanged.

![Figure 1. Soil-water characteristic curve for a) Minco silt-steel interface [18] and b) soil-geotextile interface [19]](image)

Fig. 3 shows the effect of net normal stress on behavior, in the constant suction of 100 kPa. It can be seen that by increasing the net normal stress from 105 to 210 kPa, the maximum and the post peak shear strength are increased and the dilative behavior is decreased. Eventually, in Fig. 4, low amount of suction caused the interface to show quite contractive behavior even though, it has the same amount of density as the other tests ($e=0.466$). The present model is able to appropriately simulate tests shown in Figs. 2-4 using parameters given in Table 2.

The second set of experimental data are those of Khoury et al. [19], who considered the behavior of unsaturated soil-geotextile interfaces, by using the same method as Hamid and Miller [18].
Figure 2. The model prediction versus the experimental data on a rough Minco silt-steel interface with $\sigma_{net}=140$ kPa, (a),(c) $s=50$ kPa and (b),(d) $s=100$ kPa, Experimental data taken from [18].

Figure 3. The model prediction versus the experimental data on a rough Minco silt-steel interface with $s=100$ kPa, and $\sigma_{net}=105, 210$ kPa, Experimental data taken from [18].

Figure 4. The model prediction versus the experimental data on a rough Minco silt-steel interface with $s=20$ kPa and $\sigma_{net}=105$ kPa, Experimental data taken from [18].
soil was a mixture of 75% sil-co-sil 250 and 25% Glass beads, size BT-9. The mixture was non-plastic with, mean diameter \((D_{50})\), 0.071 mm. Khoury et al. [19] used woven polypropylene (PP) geotextile as geosynthetic material, with diameter 100 mm attached to a stainless cylindrical steel block. The test was conducted at initial degree of saturation of 0.67 and initial void ratio is estimated 0.67. Fig. 1(b) shows the SWCC curve related to this set of data. The model parameters used in simulation of experiments reported by Khoury et al. [19] are presented in Table 3. Besides, \(t=1\) mm is assumed in all simulations. Effects of suction and net normal stress on the behavior of unsaturated soil-geotextile interface are studied in Figs 5 and 6, respectively. In all cases, the model can well simulate the behavior of unsaturated soil-geotextile interfaces.

Table 2: The model parameters for unsaturated soil-geotextile interfaces

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(K_0^0)</th>
<th>(K_0^b)</th>
<th>(c_0)</th>
<th>(\lambda)</th>
<th>(M)</th>
<th>(a)</th>
<th>(b)</th>
<th>(m)</th>
<th>(n)</th>
<th>(A_0)</th>
<th>(A_1)</th>
<th>(b_0)</th>
<th>(S_0)</th>
<th>(\Omega)</th>
<th>(a_0)</th>
<th>(n_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>320</td>
<td>350</td>
<td>0.83</td>
<td>0.81</td>
<td>0.65</td>
<td>0.5</td>
<td>0.7</td>
<td>0.2</td>
<td>1.5</td>
<td>0.1</td>
<td>0.8</td>
<td>1.4</td>
<td>0.01</td>
<td>0.3</td>
<td>0.15</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Figure 5. The model prediction versus the experimental data on a soil-geotextile interface with \(\sigma_{net}=100\) kPa, (a),(c) \(s=25\) kPa and (b),(d) \(s=50\) kPa, Experimental data taken from [19].

Figure 6. The model prediction versus the experimental data on a soil-geotextile interface with \(s=100\) kPa, and \(\sigma_{net}=50, 100\) and 300 kPa, Experimental data taken from [19].
5. **CONCLUSIONS**

A constitutive model for unsaturated soil-structure interfaces was presented. By using the modified effective stress concept and SWCC curve, an appropriate hydro-mechanical coupling was conducted. By comparing the model predictions with experimental results reported by Hamid and Miller [18] and Khoury et al. [19], performance of the model was examined. It was shown that the model is able to simulate the important features of unsaturated soil-structure interfaces without the need to change the model parameters.

6. **REFERENCES**


