Investigation of the stochastic dynamics of nanomotor protein

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Abstract

The stochastic dynamics of nanomotor based on Langevin equation using the bistable potential in its symmetric and asymmetric forms in presence of thermal noise is investigated. The asymmetric bistable potentials have two minima which have been separated by potential barrier with two different coupling strengths. The effect of both symmetric and asymmetric bistable potentials on the stationary probability density and the mean current is studied.

Introduction

The molecular nanomotors are of a great interest to both physicist and biologist and are described in several papers which appeared in the last decades [1-2]. In this paper we study the motion of the nanomotor as two Brownian particles coupled by a bistable potential on a periodically rocket ratchet.

Theory

In the present work we consider the asymmetric hand-over-hand model [3]. This model considers a one-dimensional walker with two feet on an asymmetric ratchet potential mediated by noise. x and y are the positions of the two feet that are represented as two particles coupled nonlinearly through a bistable potential. The stochastic differential equations for the two particles are given by the overdamped Langevin equations:

\[
\begin{align*}
x &= -\frac{\partial U(x, y, t)}{\partial x} + 2D\zeta_1(t), \\
y &= -\frac{\partial U(x, y, t)}{\partial y} + 2D\zeta_2(t)
\end{align*}
\]

The parameter D is the intensity of the zero-mean statistically independent Gaussian white noise \(\zeta_1(t)\) and \(\zeta_2(t)\). It can be shown that the potential \(U(r, s, t)\) is reduced to the following equation:
\[ U(r, s, t) = -\frac{1}{\pi} \sin(\pi r) + \frac{1}{\pi} \sin(4\pi s) \cos(2\pi r) + 2F(t)s + w(r) \]  

The symmetric potential \( w(x - y) \) has the following expressions:

\[ w(x - y) = -\frac{1}{2} \left( \frac{x - y}{r_0} \right)^2 \left( 1 - \frac{x - y}{r_0} \right)^{2\alpha} \quad \alpha = 1, 2, 3, \ldots \]  

where \( r_0 \) is the distance of the two minima of the potential. The asymmetric potential is introduced as:

\[ w(x - y) = \begin{cases} 
\frac{1}{8} \left( \frac{x - y}{r_0} \right)^2 \left( \frac{x - y}{r_0} \right)^{2\alpha} & \text{for } x - y > 0 \\
\frac{1}{8} \left( \frac{x - y}{r_0} \right)^2 \left( \frac{x - y}{r_0} \right)^{2\alpha} & \text{for } x - y < 0 
\end{cases} \]  

In our calculations the current has been obtained from the following expression:

\[ J = \langle s >_{s,t} = \int_{-r_0}^{r_0} < s >_{s,t} \rho(F) dF \]  

Results

The obtained diagrams of potential and current have been presented in the Figs. 1 and 2. Comparison between Figs. 1a and 1a, demonstrates that there is not any main difference between symmetric and asymmetric forms of bistable potentials i.e. symmetry of bistable potential does not play important role in the form of potential. Figs. 2a and 2b show current \( J \) vs. noise intensity \( D \), for various values of coupling strength \( \eta \). As Fig. 2a shows as the noise intensity increases, the current increases.
Figure 1. The potential for $F_0 = 0$, $r_0 = \frac{1}{1}$ and $D = 0.2$, $a=1/3$ using the symmetric bistable potential (a: right side) and asymmetric bistable potential (b: left side).
For low noise intensity, the current for all values of \( \eta \) is the same, but for high noise intensity, as \( \eta \) increases, current decreases. We expect this behavior to be similar to what has been reported for the other bistable potentials. Figs. 2b depicts that for all values of noise intensity D except from the small ones, the current is constant.

![Figure 2. Current J vs. noise intensity D for F=0.1 and \( r_0=1 \) using the symmetric bistable potential (a: right side) for \( a_1 = 2 \) and \( a_2 = 2 \) (box), \( a_2 = 1 \) (cross), \( a_2 = 2/3 \) (diamond) and \( a_2 = 1/2 \) (circle). Using the asymmetric bistable potential (b: left side) for \( a_1 = 1/3 \) and \( a_2 = 1/5 \) (box), \( a_2 = 1/7 \) (cross), \( a_2 = 1/9 \) (diamond) and \( a_2 = 1/11 \) (circle).](image)

References